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A FILTER TECHNIQUE FOR STOCHASTIC COOLING OF BETATRON OSCILLATIONS

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Stochastic cooling has been demonstrated (1) for both betatron oscillations and momentum spread. For momentum cooling, a novel technique was developed (2) by which the signal from a gap pick-up electrode is filtered to remove both signal and noise for a chosen value of momentum. In this way the signal/noise ratio is greatly enhanced in the cooling process, resulting in faster momentum cooling. Momentum cooling times of ~ 2 sec have been observed for a beam of $\sim 10^7$ protons.

I describe in this paper a possible filter technique for betatron stochastic cooling. The technique has three main features: 1) Betatron amplitude is coupled to betatron tune using a family of octupoles; 2) a suitable filter has been invented for notching the frequency spectrum for small-amplitude betatron motion; 3) the filter is adapted to a betatron pickup/kicker system to accomodate stochastic cooling.

Betatron Motion and Detection

Fig. 1. illustrates a split electrode suitable for detecting transverse betatron motion in the precooler. The feed-back system responds to the dipole moment $\underset{\sim}{d}$ of the beam current, with a transfer function $G(\omega)$. For a single particle with betatron amplitude A and phase ϕ, the dipole moment (current x displacement)

$$d(t) = A e f_0 \exp j(\omega_\beta t - \theta) \sum_n \exp j(n\omega_0 t - \phi)$$

where f_0 and ϕ are the revolution frequency and phase, and ω_β is the betatron frequency. This corresponds to a transform

$$d(\omega) = A e \omega_0 \exp[-j(\theta+\phi)] \sum_{n\pm} \delta(\omega - n\omega_0 \mp \omega_\beta)$$

Assume that the feedback system operates in a bandwidth W, containing $n_0 = W/f_0$ revolution harmonics. The correction at the kicker for one particle is

$$\Delta A_c = -A \cos\phi \sum_{n\pm} \mathrm{Re}\, G(n\omega_0 \pm \omega_\beta)$$

Further assume that the transfer function G of the filter is the same near each of the frequencies $n\omega_0 \pm \omega_\beta$:

$$G(n\omega_0 \pm \omega_\beta) \equiv G(\omega_\beta - \omega_{\beta 0})$$

where $\omega_{\beta 0}$ is the small amplitude betatron frequency. Then

$$\Delta A_c = -A\, n_0 \mathrm{Re}\, G/\sqrt{2}$$

The noise at the amplifier input has two components: the incoherent heating from the N other particles in the sample; and the flat spectrum of amplifier noise. The mean-squared dipole moment in the sample is

$$\langle d^2 \rangle = \frac{N}{2} n_0 (A e f_0)^2$$

The heating term in betatron amplitude is then

$$\Delta(A^2) = 2A\Delta A_h = A^2 \left(\frac{N}{2}\right)\left[\frac{\omega_0}{\Omega} + U\right] |G|^2$$

where $\Omega^{-1} = \frac{1}{N} \sum^{N} |\omega_\beta - \omega_{\beta 0}|^{-1}$, and $U = \left\langle \frac{\text{noise power}}{\text{signal power}} \right\rangle$

$$\Delta A_h = \frac{1}{4} A N n_o |G|^2 \left(\frac{\omega_o}{\Omega} + U\right)$$

This leads to a time evolution

$$\frac{1}{A}\frac{dA}{dt} = \frac{f_o}{A}(\Delta A_h + \Delta A_c) = \frac{W}{2N}\left[2g - g^2\left(\frac{\omega_o}{\Omega} + U\right)\right]$$

$$g = GN/\sqrt{2}$$

The Betatron Filter

In order to filter betatron signals in the desired manner, it is necessary to 1) induce a spread in betatron tune proportional to amplitude; 2) suppress any spread in revolution requency; and 3) invent a filter to notch the betatron sidebands with the proper phase characteristic.

To achieve condition 1, we introduce a family of octupoles in the storage ring lattice, with strength A_4 (relative to guide field). The tune is then

$$\nu_z = \nu_{zo} - 3\frac{e}{\rho_o^2}\langle\beta_z^2 A_4\rangle \varepsilon_z + 6\frac{e}{\rho_o^2}\langle\beta_x\beta_z A_4\rangle \varepsilon_x$$

If the octupole locations are chosen so that $\beta_x << \beta_z$, the cross term can be made negligible. Thus

$$\nu_z \simeq \nu_{zo} - 3\frac{e}{\rho_o^2}\langle\beta_z^2 A_4\rangle \varepsilon_z = \nu_{zo} - 3\frac{e}{\rho_o^2}\langle A_4 \beta_z\rangle \frac{A^2}{\pi}$$

The maximum tune spread that can be achieved without resonance crossing is likely to be $\Delta\nu \approx .05$.

To achieve condition 2, we assume that the storage ring is operated near transition during betatron cooling. This corresponds to the requirement

$$n_o \Delta\omega_o = \omega_o n_o \eta \Delta p/p << \Delta\omega_\beta = \omega_o \Delta\nu$$
$$n_o \eta \Delta p/p << \Delta\nu$$

For $n_0=200$, $\Delta\nu=.05$, $\Delta p/p= .02$, we must require $\eta<<.01$.

In order to notch the betatron sidebands, I have invented the filter shown in Figure 2. It incorporates single-sideband mixers, which separate the sum and difference sidebands in an RF-LO mix. This separation is crucial, since it is necessary to invert the phase characteristic between sum and difference. In each separated line, the two sideband patterns are individually filtered by shorted-line filters of the same type as are used for momentum cooling. In line 1, the sum sidebands are notched, and the difference sidebands are killed by a wide double notch. In line 2, the difference sidebands are notched, and the sum sidebands are killed. Then a final mix is done to restore the spectrum to its original basis, and the two lines summed out of phase,

Stochastic Cooling Rate

We can now calculate a cooling rate for betatron motion with filter feedback. The optimum cooling rate occurs for a gain $g_0= \frac{\omega_0}{\Omega} +U^{-1}$. The mixing term is $\omega_0/\Omega \approx (2\Delta\nu)^{-1}$, and the flat noise spectrum is suppressed by the filter. The cooling rate is

$$\frac{1}{\tau} = -\frac{dA}{A\,dt} = \frac{W}{N}\Delta\nu$$

For W=200 MHz, $\Delta\nu=.05$, $N=10^8$, we obtain $\tau= 10$ sec. This cooling time can be further reduced (as with filter momentum cooling) by using arrays of pickups rather than a single sample.

REFERENCES

1. G. Carron et al., "Experiments on Stochastic Cooling in Ice", CERN-EP/79-16, "1979".
2. L. Thorndahl, "Stochastic Cooling of Momentum Spread by Filter Technique in the Cooling Ring", CERN ISR-RF/LT/PS, "1977".

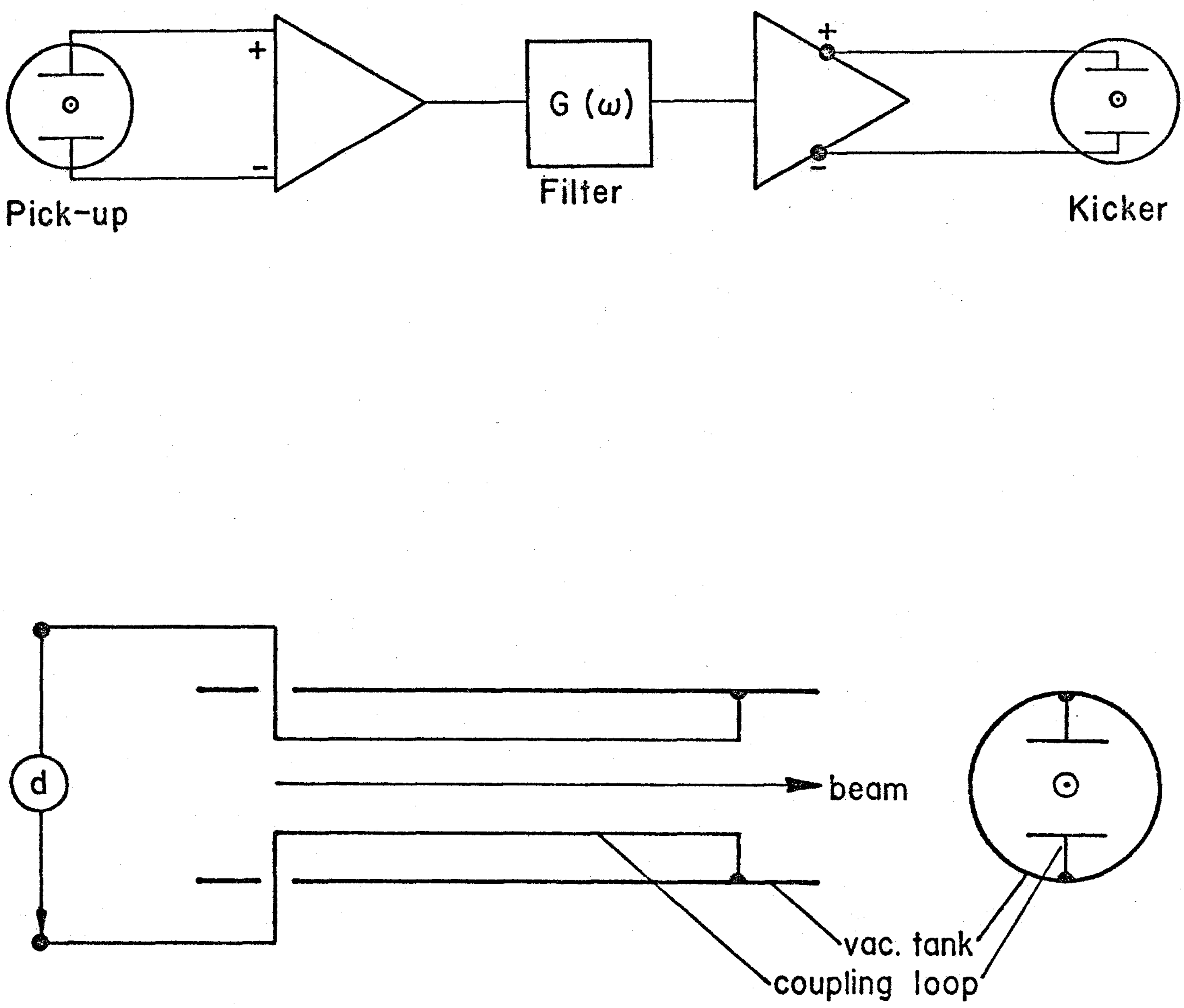

Fig. 1

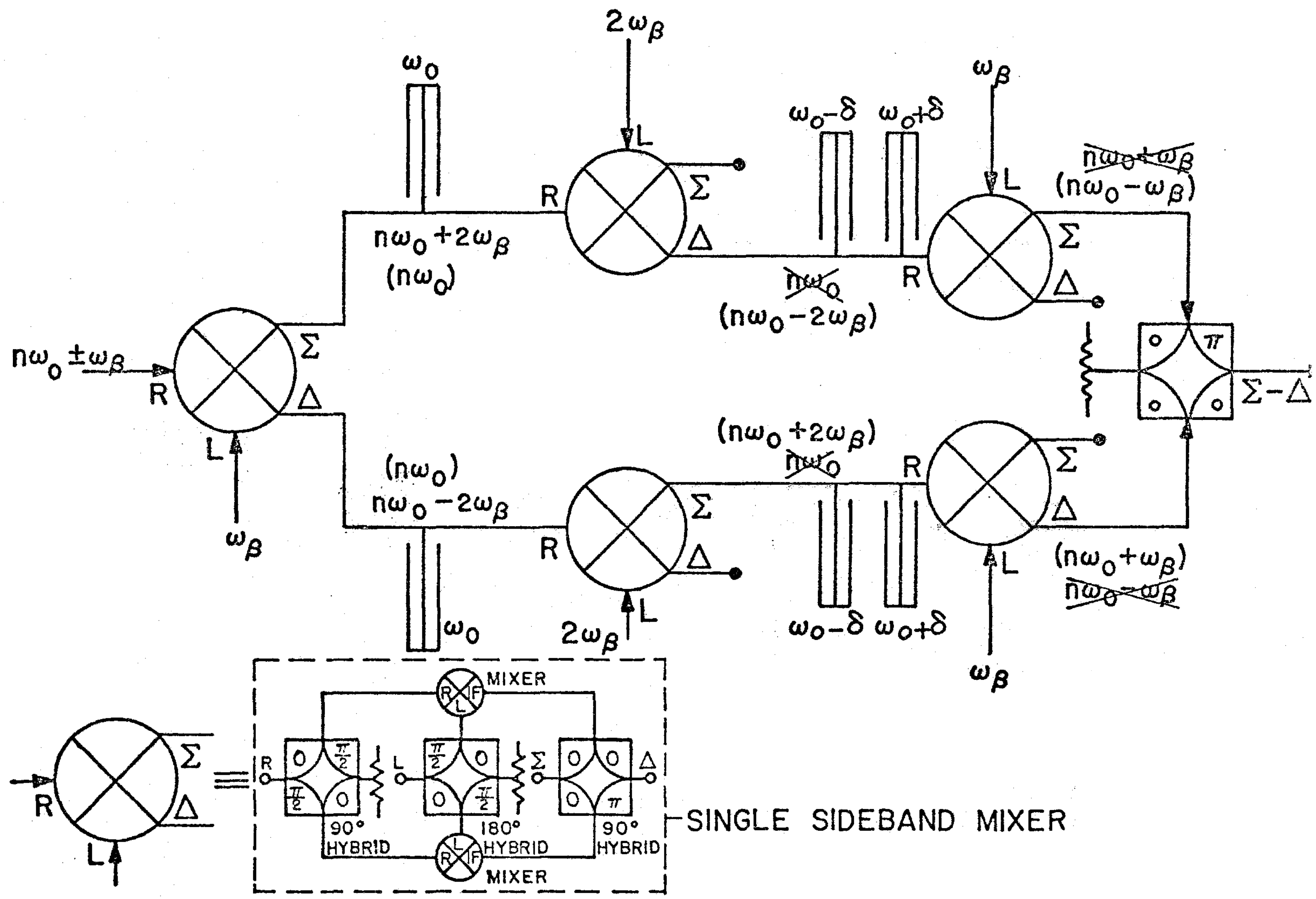
$n\omega_o \pm \omega_\beta$
R
L
ω_β
Σ
Δ
ω_o
$n\omega_o + 2\omega_\beta$
$(n\omega_o)$
$(n\omega_o)$
$n\omega_o - 2\omega_\beta$
ω_o
$2\omega_\beta$
$2\omega_\beta$
$\omega_o - \delta$
$\omega_o + \delta$
$\cancel{n\omega_o}$
$(n\omega_o - 2\omega_\beta)$
$(n\omega_o + 2\omega_\beta)$
$\cancel{n\omega_o}$
$\omega_o - \delta$ $\omega_o + \delta$
ω_β
ω_β
$\cancel{n\omega_o + \omega_\beta}$
$(n\omega_o - \omega_\beta)$
$(n\omega_o + \omega_\beta)$
$\cancel{n\omega_o - \omega_\beta}$
π
Σ-Δ
MIXER
MIXER
90° HYBRID
180° HYBRID
90° HYBRID
SINGLE SIDEBAND MIXER

FIG. 2

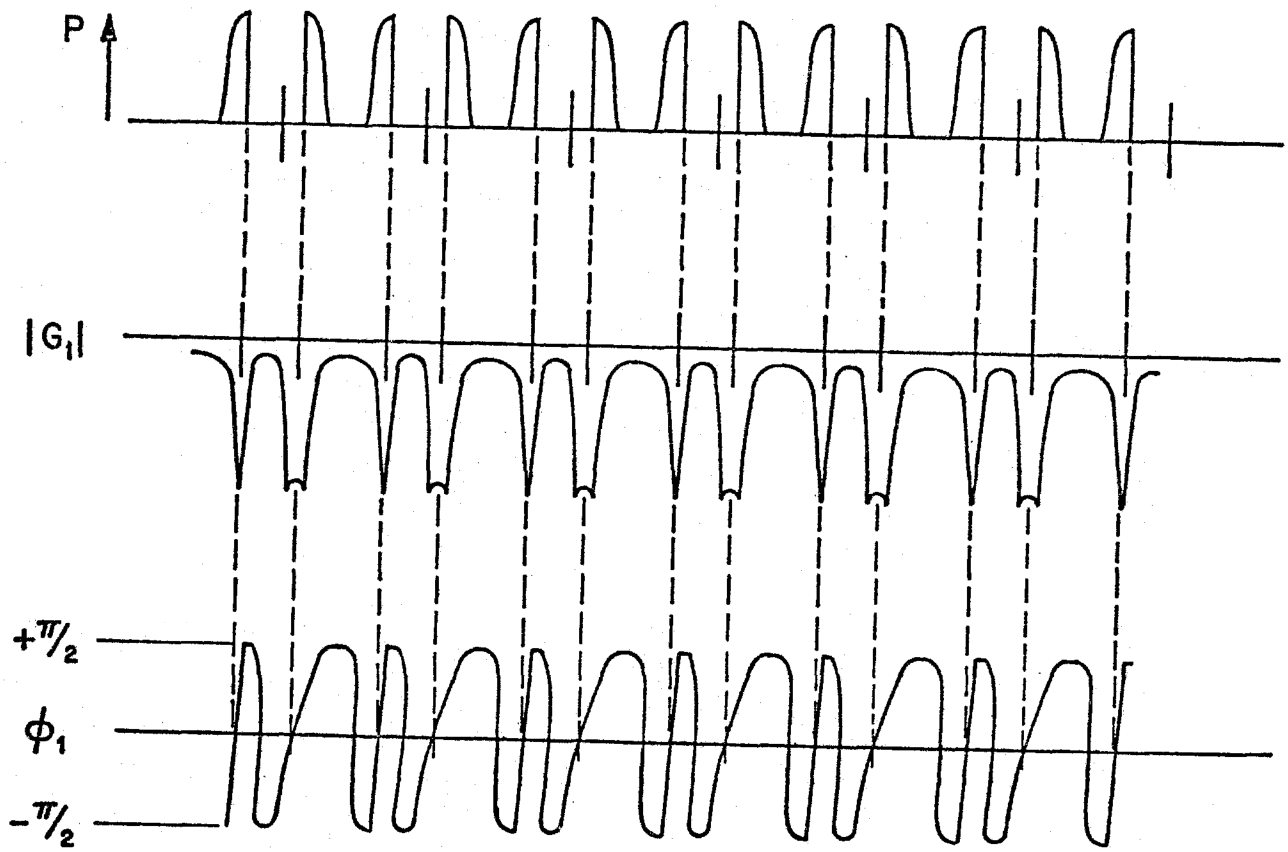

Fig. 3